On the Effect of Sovereign Risk on the International Credit Market

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Abstract

This paper investigates quantitatively the impact of sovereign risk in the stochastic stationary equilibrium of a pure exchange world economy. International borrowing and lending arise from the actions of risk averse sovereign representative agents in different countries to self-insure against idiosyncratic shocks to domestic output. The international credit market is imperfect since countries cannot commit to repay loans. The possibility of default induces countries not only to engage in additional precautionary savings, but also to restrict the amount assets available for borrowing. This work quantifies these two effects.

1 Introduction

The financial crises in emerging markets during the 90's has renewed the interest of the role of sovereign country risk in the international credit market. In a recent study, Reinhart (2002, [9]) has highlighted the statistical significance of the interaction between default and emerging markets crises. The behavior of the sovereign country risk, reflected on the interest rate that the economy faces in the international credit markets appears to be closely related to the sharp movements in the current account, the collapse in private consumption and the currency crises. The study finds that 84% of the defaults occurred in emerging markets are associated with currency crises and about 50% of the currency crises are linked to defaults. This result seems to be a particular characteristic of emerging markets since no significance is found for developed economies.

Another interesting finding is that sovereign credit ratings (used as an indicator of the likelihood of default) not only have significant impact on sovereign

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bond yield spreads, but also serve as good predictors of the occurrence of defaults (Larraín et al., 1997 [7]). In consequence, it is not surprising that during periods of financial distress lower ratings are observed and countries face more difficulties borrowing from international credit markets.

In a recent survey, Arellano and Mendoza (2002, [2]) point out that the common starting point of much of the literature on emerging markets crises has been to introduce some type of financial-market imperfection that distinguishes emerging economies from industrial countries. The majority of studies focuses on partial equilibrium models that qualitatively predict results consistent with the dynamics of the crises. Recently, the literature has shown a renewed interest on studying the role of sovereign risk in a complete markets general equilibrium framework. However, little is known about the quantitative predictions of both type of models.

Using an algorithm proposed by Huggett (1993, [6]) this paper seeks to quantify the impact of sovereign risk on the world stochastic stationary equilibrium in the context of an incomplete markets general equilibrium pure exchange model in which countries face idiosyncratic shocks to output and can default on their debts. The paper borrows the main elements from the work of Eaton and Gersovitz (1981, [4]) and Clarida (1990, [3]) to explore several questions: first, how important is sovereign risk as a precautionary savings channel? Second, what are the main determinants of the probability of default? Third, how tight is the equilibrium borrowing limit when compared to other borrowing limits used in the literature? The rest of the paper proceeds as follows: Section 2 presents the structure of the model and characterizes the equilibrium. Section 3 present the stochastic stationary equilibrium. Section 4 describes briefly the algorithm for the solution of the model. Section 5 reports the results of different experiments as well as the simulation results. Section 6 concludes.

2 The Model

Consider a world exchange economy with a continuum of countries of total mass equal to one. Each country is populated by an infinitively lived representative agent. Time is discrete, and each period each agent receives a stochastic endowment, y of a perishable consumption good. The set Y denotes the possible discrete values that the endowment can take. Each country's endowment follows a Markov process with stationary transition probability $\pi(y'|y) = \Pr(y_{t+1} = y'|y_t = y) > 0$ for $y,\, y' \in Y$ that is independent of all other countries' current and past endowments. This strong assumption is needed to rule out aggregate world uncertainty. Each country has preferences defined over consumption given by:

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$$

where $\beta \in (0,1)$ and $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. In this setting, the country can smooth consumption by holding a single asset. Countries can access the international credit market in one-period consumption loans, and are allowed to borrow from a set of loans A. The asset level must remain above a borrowing limit $\underline{a} < a$, to be determined endogenously. Notice that if $a > 0$ the country is a lender, but if $a < 0$ the country is a borrower. The country can choose to repay or default on its outstanding debt. If the country decides to repay, then the resources constraint is given by:

$$c + a' \leq y + (1 + r)a \tag{1}$$

where $c \geq 0$ and $a \geq \underline{a}$, and r is a given interest rate. If the country decides to default, it is not allowed to borrow and has to consume the endowment thereafter. So, the constraint in the case of default is $c \leq y$.

A country's position at a point in time is described by a state vector $s \in S$. $s = (y, a, d)$ indicates a country's net foreign assets a, the level of output y and the default state d. Notice that $d = 1$ is an absorbent state. The state space is $S = Y \times A \times D$, where $Y = \{y_1, ..., y_{n_1}\}$, $A = [a_1, ..., a_{n_2}]$ and $D = \{0, 1\}$.

The problem of the representative agent of a given country can be described as exercising an option to default at time t:

$$v(s; r) = \max \left\{ \max_{(c,a') \in \Gamma(s;r)} \left\{ u(c) + \beta \sum_{y'} \pi(y'|y) v(s'; r) \right\}, w(y) \right\} \tag{2}$$

where

$$\Gamma(s; r) = \{(c, a') : c \leq y + (1 + r)a - a'; c \geq 0; a' \geq \underline{a}; \}$$

and

$$w(y) = E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau - t} u(y_\tau) \right] \tag{3}$$

d' indicates whether the country has chosen to default or to repay optimally:

$$d' = \begin{cases} 0 \text{ if } \max_{(c,a') \in \Gamma(s;r)} \left\{ u(c) + \beta \sum_{y'} \pi(y'|y) v(s'; r) \right\} \geq w(y) \\ 1 \text{ otherwise} \end{cases} \tag{4}$$

Assume that a bounded measurable solution v to functional equation (2) exists and it is optimal. If v is the optimal value function, then the functions $c : S \to R_+$, $a : S \to A$ and $d : S \to D$ are the optimal decision rules provided $c(s; r)$, $a(s; r)$ and $d(s; r)$ are measurable and satisfy (2).

3 Equilibrium

Before describing formally the equilibrium concept it is useful to compare the elements of this model with respect to a model in which there is no possibility of default. In the absence of default, the equilibrium of the model is identical to Huggett (1993, [6]) and Aiyagari (1994, [1]).[1] The world per-country supply

[1] Both authors use a model of heterogeneous agents in a closed economy. Here we think as agents, being sovereign countries trading assets in the world economy.

of assets is a continuous function of the interest rate. An equilibrium exist as long as the interest rate does not exceed the discount factor. Heterogeneity implies that for a given country when the interest rate is low it would like to borrow. When the interest rate is relatively high the country would like to lend. Equilibrium occurs when the average level of assets (over countries) is zero. When there is no room for default, countries cannot accumulate debt beyond an exogenous borrowing limit (either ad-hoc or "natural"). As the interest rate becomes more negative, the expected level of assets approaches the exogenous borrowing limit.

When countries can default, there is an additional risk factor. The precautionary savings channel operates, and the net supply of assets shifts to the right. The interest rate in a world economy populated by heterogeneous countries each with the potential possibility of defaulting is lower. As the interest rate becomes more negative, the expected level of assets approaches an endogenous borrowing limit. This limit is tighter than the exogenous one, because the possibility of default limits the amount of assets that a given country will supply.

The equilibrium concept used is that one of a stochastic, stationary equilibrium represented by a time invariant interest rate and a time invariant probability distribution of economies. A stochastic stationary equilibrium is characterized by a probability measure ψ defined over subsets of the state space, $X \subset S$, and the Borel $\sigma-$algebra, β_X. So, for $B \in \beta_X$, the probability $\psi(B)$ represents the mass of countries whose state vector lies in B. Now, in this equilibrium the international interest rate, r, is also stationary since it depends on the distribution, ψ.

The stationary equilibrium is defined recursively. Given the transition rule $a(s)$, and the transition probabilities $\pi(y'|y)$, the optimal transition probability $P(s, B)$ can be computed. $P(s, B)$ is the probability that a country in state s will reach a state vector that lies in B.[2] So, the distribution ψ defined on (X, β_X) is stationary if the distribution over countries is unchanging.

Formally, a stationary equilibrium for the world economy is a set of rules $c(s), a(s)$, an interest rate, r, and a probability measure ψ satisfying:

1. $c(s)$, $a(s)$ and $d(s)$ are optimal decision rules for a given r.
2. The goods market clears: $\int_X c(s)d\psi = \int_X y d\psi$
3. The asset market clears:$\int_X a(s)d\psi = 0$.
4. ψ is a stationary probability measure: $\psi(B) = \int_X P(s, B)d\psi$ for all $B \in \beta_X$.

The first condition states that countries optimize to find a set of rules for consumption, asset accumulation and default that depend on the current state of

[2] The optimal transition probability $P(s, B)$ is very useful to compute the probability of default. Since there are transient and absorbing states, the probability of default is given by the probability of reaching an absorbing state.

their economies. The second and third conditions are the market clearing conditions. The third condition states that the distribution of countries over states is invariant.

4 Algorithm

The computational method used for the solution of the model is similar to that of Huggett (1993, [6]) and Aiyagari (1994, [1]). The only difference here is that this economy has an additional discrete state and and action: default or repay. To simplify notation, let $g(s)$ denote the policy functions $(a(s), d(s))$. The method is based on three steps:

1. For a given interest rate r compute $g(s;r)$ using equation (2) and obtain the optimal transition probability matrix $P(s, B)$.
2. Given $g(s;r)$ iterate on $\psi^{(i+1)}(B) = \int_X P(s,B)d\psi^{(i)}$ from an arbitrary initial distribution $\psi^{(0)}$. After convergence of the sequence $\{\psi^{(i)}\}$, use the resulting distribution ψ to compute the expected level of assets: $\int_X a(s;r)d\psi$.
3. Update r and repeat steps 1 and 2 until the market clearing condition is approximately satisfied, $\int_X a(s;r)d\psi = 0$.
4. In addition, the borrowing limit can be computed as the level of assets, $\underline{a}$ such that $\lim_{r\to-\infty} \int_X a(s;r)d\psi$.

The first step of the algorithm is solved by policy function iteration on equation (2), for a given $r^{(i)}$. The state space S is discretized in a grid of 1200 equally points. Output is discretized following Tauchen and Hussey (1991, [10]) and the asset levels are discretized by an equally spaced grid, with an ad-hoc debt limit of -3. The default/repay state is discrete by definition.

The second step requires to iterate on $\psi^{(i+1)}(B) = \int_X P(s,B)d\psi^{(i)}$ from an arbitrary initial distribution $\psi^{(0)}$. To compute the ergodic distribution notice that the optimal policies $a(s), d(s)$ and the Markov chain π on y induce a Markov chain on s, $P(s, B)$ via the formula:

$$\Pr[s_{t+1} = s'|s_t = s] = \Pr[d_{t+1} = d'|s_t = s]\Pr[a_{t+1} = a'|s_t = s]\Pr[y_{t+1} = y'|y_t = y]$$

or:

$$\Pr[s_{t+1} = s'|s_t = s] = \iota(d', a', s)\pi(y, y') \tag{5}$$

where $\iota(d', a', s) = 1$ if $a' = a(s)$ and $d' = d(s)$ and 0 otherwise. This indicator function identifies the time t states s that are sent into (a', d') at time $t+1$. Equation (5) defines an $n \times n$ matrix P where n is the number of total possible states. The matrix P is used to compute the ergodic distribution, ψ and the probability of default.

Before continuing to step 3, the computation of the probability of default deserves some comment. Typically, when there is no default option, all the states of the Markov Chain associated with P will be recurrent and its stationary distribution, ψ, can be interpreted as the fraction of time that the country spends in each state. When the possibility of default exists there are two types of states: repayment states, which are transient and defaulting states which are recurrent (once one of the states of this set of states has been reached, the state of the system moves only among them). The numerical results show that the matrix P has the following structure:

$$P = \begin{bmatrix} Q & \lambda \\ 0 & \pi \end{bmatrix}$$

where Q is a matrix that yield the probability of moving from a transient state (repayment) to a transient state (repayment) in the next period, λ is a matrix that yield the probability of moving from a repayment state to an defaulting state in the next period, and π yields the probability of staying in the recurrent states. Note that once the decision of default has been taken there is no possibility to reach a repayment (transient) state in the next period. The optimal probability of default in period $t+1$ at a given time t can be computed as[3]:

$$\lambda_{t+1} = \sum_{\tau=0}^{t} Q^{t-\tau} \lambda \pi^{\tau} \tag{6}$$

where λ is the probability of default at $t = 1$.

Step 3 updates r , using the market clearing condition:

$$\sum_{s} \psi^{(i)} a(s; r^{(i)}) = e^{(i)}$$

If $e^{(i)} > 0$, then increase the interest rate to $r^{(i+1)} > r^{(i)}$ and use $r^{(i+1)}$ as a new interest rate. Continue iterating on steps 1 and 2 until $e^{(i)} = 0$. This is actually a root-finding problem. The solution technique used is the bisection method.[4] The extremes values of r are chosen in such a way that the negative value of r is large enough to compute the borrowing limit. Positive values of r require to be strictly less than $(1-\beta)/\beta$ for the equilibrium to be well defined.

5 Results

The parameters used for the solution of the model are the discount factor $\beta = 0.96$, the coefficient of relative risk aversion 2. The mean of output is set at 1. For the benchmark model the volatility of the shocks, measured by the standard deviation of the output process is set at $\sigma = 0.4$. The persistence of the shocks, measured by the autocorrelation coefficient is set at $\rho = 0.2$. Sensitivity analysis is performed on other values of the parameters of the stochastic output process.

[3]For a proof of this result, see Medhi (1994, [8]) pages 116-117.

[4]For a practical exposition of the method, see Fackler and Miranda (2002, [5])

Figure 1 compares the expected assets, as a function of the interest rate, of a "pure credit" model in which there is no possibility of default and the expected asset function of a model in which default is a potential threat. For the benchmark parameterization, in the pure credit model the equilibrium interest rate is about 3% and the ad-hoc borrowing limit goes to -3. With default risk, the equilibrium interest rate is about 1% and the endogenous borrowing limit tends to -1. Considering that the output mean is set at 1, the effect of sovereign risk on the tightening of the borrowing limit can be significant. The impact of sovereign risk on the world equilibrium interest rate is also significant (2 percent points).

It is interesting to note that the stochastic properties of the stationary equilibrium in the model with potential default are qualitatively similar to the model without default, but quantitatively very different. The shape of the expected assets curve is similar to that obtained in numerical simulations by Aiyagari (1994, [1]). However, for negative values of assets (positive debt) the expected assets curve is steeper and approaches faster the borrowing limit.

All the equilibrium outcomes in this model are influenced by the incentives that the countries have to default. In turn, these incentives depend on the preferences of the countries (which are assumed to be identical for all), the level of debt of a particular country and the nature of the idiosyncratic shocks. Eaton and Gersovitz (1981, [4]) has shown that, for a given country, the higher the risk aversion, the higher the inter-temporal discount rate, the higher the level of debt and the lower the output of a country the higher is the probability of default of a particular country. In a world economy in which countries are heterogeneous, the default incentives will depend on the interaction of all these factors represented on the equilibrium interest rate and the borrowing limit. The next experiments illustrate the sensitivity of the equilibrium interest rate, the borrowing limit and the probability of default to the volatility and persistence of the stochastic output process, for a given discount factor and relative risk aversion coefficient.

Table 1: Equilibrium Interest Rate

ρ/σ	0.1	0.2	0.3
0.0	0.0361	0.0364	0.0351
0.3	0.0361	0.0366	0.0265
0.6	0.0362	0.0252	0.0128
0.9	0.0333	0.0209	0.0051

Table 1 shows how the equilibrium interest rate reacts to changes in the stochastic properties of output. In general, increased volatility and persistence reduce the equilibrium interest rate. This is because the typical effect of increased precautionary savings found in Aiyagari (1994, [1]).

Table 2: Endogenous Borrowing Limit

ρ/σ	0.1	0.2	0.3
0.0	-2.5000	-2.2657	-2.1100
0.3	-2.5000	-2.1697	-2.0350
0.6	-2.5000	-2.2300	-1.8560
0.9	-2.5000	-2.2200	-1.6705

Table 2 shows how the borrowing limit changes as the volatility and persistence of output change. Higher volatility of output tightens the borrowing limit. This result arises from precautionary savings and incomplete markets. Except in the case in which the volatility is relatively low, increased persistence also tightens the borrowing limit.

Table 3: Maximum Probability of Default

ρ/σ	0.1	0.2	0.3
0.0	0.1412	0.1986	0.2420
0.3	0.1986	0.2033	0.2427
0.6	0.3309	0.2680	0.2671
0.9	0.5215	0.5304	0.5400

Table 3 shows how the maximum (over the state space S) of the one-period ahead probability of default changes as the stochastic properties of output change. Increased volatility and persistence raise the equilibrium probability of default. This result is a natural outcome of precautionary savings under incomplete markets. Increased uncertainty raise the value of having access to international credit markets, but at the same time raises the negative effect on current utility of an adverse shock. Since contracts are not state contingent, the probability of default is higher when the shocks are potentially more adverse.

6 Conclusions

This paper investigates quantitatively the impact of sovereign risk in the stochastic stationary equilibrium of a pure exchange world economy. The main conclusion is that sovereign risk appears to have a significant impact on the borrowing limits that the economies face, and on the equilibrium real interest rate. Since idiosyncratic shocks to output are not insurable, sovereign risk increase the precautionary savings motive for the countries. Expected assets holdings increase, since this is the only mechanism that countries can use to save, and so the real interest rate that clears the world asset market is substantially lower.

The impact of the stochastic properties of output on the equilibrium of the model are also analyzed. The numerical results show that increased volatility and persistence of shocks induce tighter borrowing limits, larger probabilities of default and lower world equilibrium real interest rates.

In further research it would be interesting to study some features that were abstracted from this model. A natural extension is to provide the countries an additional savings channel by introducing capital accumulation. With capital as an additional mean of savings one may suspect that the precautionary savings effect of sovereign risk may be lower. There are other interesting features to analyze like differences in the maturity of the debt contracts, aggregate world uncertainty, as well as other credit market imperfections.

References

[1] S. Rao Aiyagari. Uninsured idiosyncratic risk and aggregate savings. *Quarterly Journal of Economics*, pages 659–684, August 1994.

[2] Cristina Arellano and Enrique Mendoza. Credit frictions and sudden stops in small open economies: An equilibrium business cycles framework for emerging markets crises. Working Paper 8880, National Bureau of Economic Research, 2002.

[3] Richard Clarida. International borrowing and lending in a stochastic stationary equilibrium. *International Economic Review*, 31:543–558, 1990.

[4] Jonathan Eaton and Mark Gersovitz. Debt with potential repudiation: Theoretical and empirical analysis. *Review of Economic Studies*, 48:289–309, 1981.

[5] Paul Fackler and Mario Miranda. *Applied Computational Economics and Finance*. MIT Press, forthcomming, 2002.

[6] Mark Huggett. The risk-free rate in heterogeneous-agent economies. *Journal of Economic Dynamics and Control*, 17:953–969, 1993.

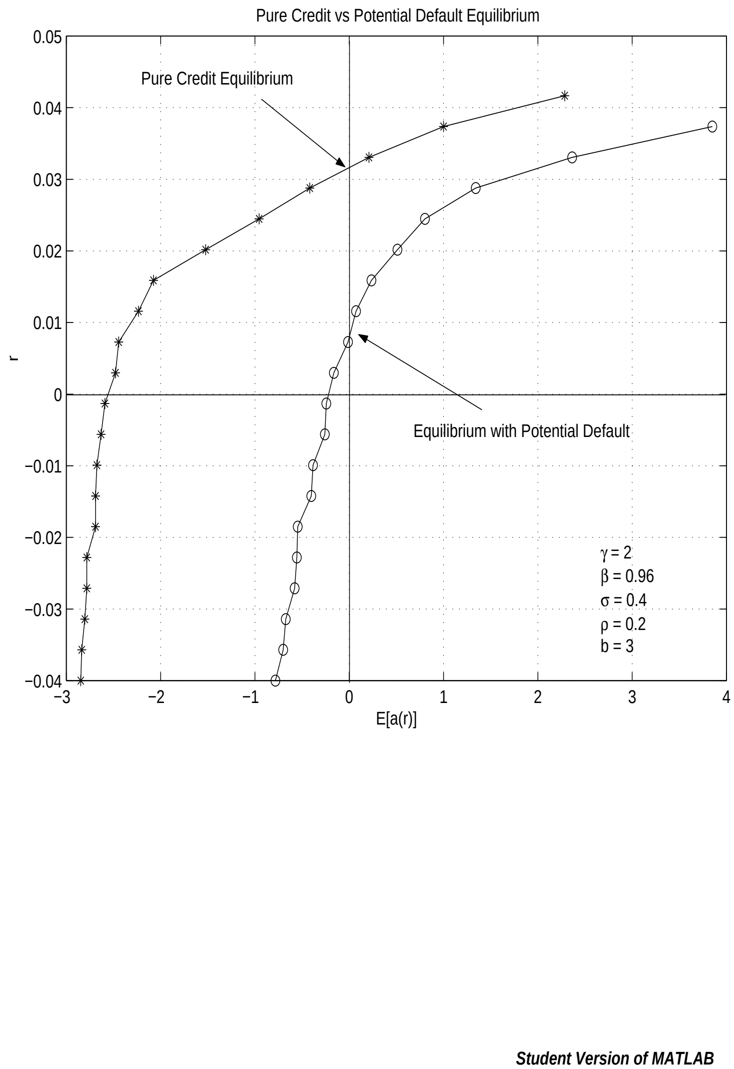
[7] Guillermo Larrain, Helmut Reisen, and Julia von Maltzan. Emerging market risk and sovereign credit ratings. Working Paper 124, OECD Development Centre, April 1997.

[8] Jyotiprasad Medhi. *Stochastic Processes*. John Wiley and Sons, 1994.

[9] Carmen M. Reinhart. Default currency crises and sovereign credit ratings. Working Paper 8738, National Bureau of Economic Research, 2002.

[10] George Tauchen and Robert Hussey. Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models. *Econometrica*, 59:371–396, 1991.

Figure 1


Pure Credit vs Potential Default Equilibrium
Pure Credit Equilibrium
Equilibrium with Potential Default
γ = 2
β = 0.96
σ = 0.4
ρ = 0.2
b = 3
r
E[a(r)]
0.05
0.04
0.03
0.02
0.01
0
−0.01
−0.02
−0.03
−0.04
−3
−2
−1
0
1
2
3
4
Student Version of MATLAB