A Note on the Identification of Preferences from the Equlibrium Manifold under Complete Markets

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Abstract

We prove that the observability of the equilibrium manifold of an exchange economy is sufficient to globally recover (in prices and individual endowments) individual demands. We extend previous local results for the same problem (see Chiappori, Ekeland, Kubler and Polemarchakis [2000]).

1 Introduction

The transfer paradox, first pointed out by Leontief (1936), and generalized by Donsimoni and Polemarchakis (1994), illustrates the importance of identifying the fundamentals of an economy from observable data. Under the hypothesis of general equilibrium, the aggregate demand function cannot be assumed to be observed: at equilibrium prices aggregate demand is, by definition, equal to aggregate endowment. Demand, either individual or aggregate, cannot be observed for out-of-equilibrium prices. One can observe, however, equilibrium prices and individual incomes. In this paper we address the problem of identifying individual preferences from the equilibrium manifold when asset markets are complete.

For the case of complete markets, positive results have been obtained by Balasko [1999], Chiappori et al [2000] and Matzkin [2003]. Balasko's result has been criticized for making very strong observational assumptions: that one can observe equilibrium prices in situations in which endowment is zero for all individuals but one. Under additional assumptions, Chiappori et al obtain local identification of individual demands using a constructive argument. Matzkin determines the largest class of fundamentals for which identification is possible. Her argument, however, is not constructive.

We identify individual demands globally. When we have generically complete real assets structures, we conjecture that our results hold generically on prices and endowments. We use Balasko's idea on how to recover the aggregate demand function from the equilibrium manifold, hence we avoid using the implicit function theorem. We then use a slightly different argument than Chiapori et al. to identify individual demands from the aggregate demand function and we also avoid using Balasko's strong observational assumption pointed out before.

2 The Model

Consider a standard exchange economy $\xi = \left(\{u_i\}_{i\in\mathcal{I}}, \{w_i\}_{i\in\mathcal{I}}\right)$ where $\mathcal{I}$ is a finite set of agents, $\mathcal{I} = \{1,...,I\}$, u_i are agents utility functions over $\mathbb{R}^L_+$ and w_i are agents endowments where $w_i \in \mathbb{R}^L_{++}$ for all $i \in \mathcal{I}$.

Condition 1 *For each $i \in \mathcal{I}$ assume u^i is continuous, and that it represents locally nonsatiate and strictly convex preferences.*

Condition 2 *For each $i \in \mathcal{I}$ and for all $x \in \mathbb{R}^L_{++}$,*

$$\left\{x' \in \mathbb{R}^L_+ : u^i(x') \geq u^i(x)\right\} \subseteq \mathbb{R}^L_{++}.$$

Let $P \in S^{L-1}_{++} = \{P' \in \mathbb{R}^L_{++} : P'_1 = 1\}$ be the vector of normalized prices of the L commoditties, and for each $i \in \mathcal{I}$ define agents budget constraint as,

$$B\left(P, \omega^i\right) = \{x \in \mathbb{R}^L_+ : P \cdot x \leq P \cdot \omega^i\}$$

Condition 3 *For each $i \in \mathcal{I}$ assume u^i is continuously differentiable in $\mathbb{R}^L_{++}$.*

Definition 4 *For each $i \in \mathcal{I}$, define the individual demand function $f^i : S^{L-1}_{++} \times \mathbb{R}^L_+ \to \mathbb{R}^L_{++}$, as $f^i\left(P, \omega^i\right) = \arg\max\left\{u^i(x) : x \in B\left(P, \omega^i\right)\right\}$ and the aggregate demand function $F : S^{L-1}_{++} \times \mathbb{R}^{LI}_{++}$ as $F(P, \omega) = \sum_{i=1}^{I} f^i\left(P, \omega^i\right)$.*

Now, consider the dual of the maximization problem in definition 4. For each $i \in \mathcal{I}$, let $U^i = \{\mu \in \mathbb{R}_{++} : \exists x \in \mathbb{R}^L_+, u^i(x) \geq \mu\}$.

Definition 5 *For each $i \in \mathcal{I}$, define the individual hicksian demand function $h^i : S^{L-1}_{++} \times U^i \to \mathbb{R}^L_{++}$, as $h^i(P, \mu) = \arg\min\left\{P \cdot x : u^i(x) \geq \mu\right\}$*

Proposition 6 . *Let $(P, w) \in S^{L-1}_{++} \times \mathbb{R}^L_{++}$ and $\mu = u^i(f^i(P, w))$. Then, $h : S^{L-1}_{++} \times U \longrightarrow \mathbb{R}^L_{++}$ is differentiable in prices and for all $l, l' \in (\{1, ..., L\}) \setminus \{1\}$, we have:*

$$\frac{\partial h^i_l(P, \mu)}{\partial P_{l'}} = \frac{\partial f^i_l(P, w)}{\partial P_{l'}} + \frac{\partial f^i_l(P, w)}{\partial w_1}(f^i_{l'}(P, w) - w_{l'})$$

Proof. See proposition 3.G.3, page 71. Mas-Collel, Whinston and Green [1995] ■

Definition 7 *The equilibrium manifold M is:*

$$M = \left\{ (P, w) \in S_{++}^{L-1} \times \mathbb{R}_{+}^{LI} : F(P, w) = \sum_{i=1}^{I} w^i \right\}$$

Henceforth we assume that there is an economy $\xi = \left(\{u_i\}_{i \in \mathcal{I}}, \{w_i\}_{i \in \mathcal{I}} \right)$ that satisfy our assumptions. We study whether from the equilibrium manifold M, the unobserved fundamentals (i.e preferences) can be uniquely determined. We do not test the existence of such an economy (under the equilibrium hypothesis). In the following section we show that the equilibrium manifold uniquely determines aggregate demand globally. Then, we show that aggregate demand uniquely determines individual demands globally. Our arguments are constructive. That is, we prove global recoverability of individual demands from the equilibrium manifold.

3 From the Equilibrium Manifold to the Aggregate Demand

Theorem 8 *For each $(P, w) \in S_{++}^{L-1} \times \mathbb{R}_{+}^{LI}$, let $\left(\widehat{w}^i\right)_{i=1,\ldots I} \in \mathbb{R}_{+}^{LI}$ be such that*

1. $\left(P, \left(\widehat{w}^i\right)_{i=1,\ldots I}\right) \in M$
2. $P \cdot \widehat{w}^i = P \cdot w^i$ *for all i.*

We define aggregate demand as:

$$F(P, w) = \sum_{i=1}^{I} \widehat{w}^i$$

Proof. (Balasko [1999]). We first prove that the aggregate demand is well defined. There is at least one $\left(\widehat{w}^i\right)_{i=1,\ldots I} \in \mathbb{R}_{+}^{LI}$ that satisfies the previous two conditions: let $\widehat{w}^i = f^i(P, w^i)$, then $\left(P, \left(\widehat{w}^i\right)_{i=1,\ldots I}\right)$ is an equilibrium of the economy $\mathcal{E} = \left(\mathcal{I}, \left(u^i, \widehat{w}^i\right)_{i \in \mathcal{I}}\right)$ and by Walras Law, $P \cdot \widehat{w}^i = P \cdot w^i$.

Now, if $\left(P, \left(\widehat{w}^i\right)_{i=1,\ldots I}\right) \in M$, is such that $P \cdot \widehat{w}^i = P \cdot w^i$ for all i, then by the definition of equilibrium:

$$\sum_{i=1}^{I} \widehat{w}^i = \sum_{i=1}^{I} f^i(P, \widehat{w}^i)$$

and since $B\left(P, \widehat{w}^i, A\right) = B\left(P, w^i, A\right)$ then, $f^i(P, \widehat{w}^i) = f^i(P, w^i)$, which implies that $\sum_{i=1}^{I} \widehat{w}^i = \sum_{i=1}^{I} f^i(P, w^i)$. ■

Remark 9 *Notice that the previous prove only uses Walras Law.*

4 From the Aggregate Demand to the Individual Demand

If one is willing to assume that equilibrium prices are observable for situations in which the incomes of all individuals but one are zero, then it is straightforward that aggregate demand identifies individual demands: for all i, $f^i(P,w^i) = F(P,(\mathbf{0},\mathbf{0},...,w^i,...,\mathbf{0}))$. That is, when all agents different from i, have no income, the fact that prices are strictly positive implies no demand for agents different from i, and, therefore, that aggregate demand is agent i's individual demand.

We now show that under some additional assumptions one can identify an individual's demand without pegging everybody else's income at zero.

As an auxiliary result, we first show that aggregate demand identifies individual demands up to a function of prices only.

Theorem 10 *For some $\varphi^i : S^{L-1}_{++} \times \mathbb{R}^L_+ \longrightarrow \mathbb{R}^L_+$, which is identified and $\phi^i : S^{L-1}_{++} \longrightarrow \mathbb{R}^L_+$,*

$$f^i(P,w) = \varphi^i(P,w) + \phi^i(P)$$

for all $(P,w) \in S^{L-1}_{++} \times \mathbb{R}^L_+$.

Proof. Let $\varphi^i(P,w) = F(P,(\mathbf{1},\mathbf{1},...,w,...,\mathbf{1}))$, where w occupies position i of the vector. Let $\phi^i(P) = -\sum_{j=1,j\neq i}^{I} f^j(P,\mathbf{1})$ then, φ^i is identified. ■

As in Chiappori et al (2002), we impose the following:

Condition 11 (Regularidad) *For every individual i and every $P \in S^{L-1}_{++}$, there exist $w \in \mathbb{R}^L_+$, and $l,l' \in \{1,...,L\} \setminus \{1\}$, such that:*

1. $\frac{\partial^2 f^i_l}{\partial\left(w^i_1\right)^2} \neq 0$

2. $\begin{vmatrix} \frac{\partial^2 f^i_l}{\partial\left(w^i_1\right)^2}(P,w) & \frac{\partial^2 f^i_{l'}}{\partial\left(w^i_1\right)^2}(P,w) \\ \frac{\partial^3 f^i_l}{\partial\left(w^i_1\right)^3}(P,w) & \frac{\partial^3 f^i_{l'}}{\partial\left(w^i_1\right)^3}(P,w) \end{vmatrix} \neq 0$

Under regularity, global identification of individual demands is possible:

Theorem 12 *Aggregate demand identifies individual demands.*

Proof. It suffices to prove that the function ϕ^i of theorem 10 is also identified. From propositions 6, and ignoring the arguments, it follows that for every $w \in \mathbb{R}^L_+$ and $l,l' \in \{1,...,L\} \setminus \{1\}$:

$$\frac{\partial f^i_l}{\partial P_{l'}} + \left(f^i_{l'} - w^i_{l'}\right)\frac{\partial f^i_l}{\partial w^i_1} = \frac{\partial f^i_{l'}}{\partial P_l} + \left(f^i_l - w^i_l\right)\frac{\partial f^i_{l'}}{\partial w^i_1}$$

Substituting,

$$
\begin{aligned}
& \frac{\partial \varphi_l^i}{\partial P_{l'}} + \frac{\partial \phi_l^i}{\partial P_{l'}} + \left(\varphi_{l'}^i + \phi_{l'}^i - w_{l'}^i\right) \frac{\partial \varphi_l^i}{\partial w_1^i} \\
= \; & \frac{\partial \varphi_{l'}^i}{\partial P_l} + \frac{\partial \phi_{l'}^i}{\partial P_l} + \left(\varphi_l^i + \phi_l^i - w_l^i\right) \frac{\partial \varphi_{l'}^i}{\partial w_1^i}
\end{aligned}
$$

Taking that $l \neq 1$ and $l' \neq 1$ and deriving once and twice with respect to income gives us

$$
\begin{aligned}
& \frac{\partial^2 \varphi_l^i}{\partial w_1^i \partial P_{l'}} + \left(\varphi_{l'}^i + \phi_{l'}^i - w_{l'}^i\right) \frac{\partial^2 \varphi_l^i}{\partial \left(w_1^i\right)^2} \\
= \; & \frac{\partial^2 \varphi_{l'}^i}{\partial w_1^i \partial P_l} + \left(\varphi_l^i + \phi_l^i - w_l^i\right) \frac{\partial^2 \varphi_{l'}^i}{\partial \left(w_1^i\right)^2}
\end{aligned}
$$

and

$$
\begin{aligned}
& \frac{\partial^3 \varphi_l^i}{\partial \left(w_1^i\right)^2 \partial P_{l'}} + \frac{\partial \varphi_{l'}^i}{\partial w_1^i} \frac{\partial^2 \varphi_l^i}{\partial \left(w_1^i\right)^2} + \left(\varphi_{l'}^i + \phi_{l'}^i - w_{l'}^i\right) \frac{\partial^3 \varphi_l^i}{\partial \left(w_1^i\right)^3} \\
= \; & \frac{\partial^3 \varphi_{l'}^i}{\partial \left(w_1^i\right)^2 \partial P_l} + \frac{\partial \varphi_l^i}{\partial w_1^i} \frac{\partial^2 \varphi_{l'}^i}{\partial \left(w_1^i\right)^2} + \left(\varphi_l^i + \phi_l^i - w_l^i\right) \frac{\partial^3 \varphi_{l'}^i}{\partial \left(w_1^i\right)^3}
\end{aligned}
$$

We can rewrite this system as

$$
\Delta \begin{bmatrix} \phi_{l'}^i \\ \phi_l^i \end{bmatrix} = \Gamma
$$

where

$$
\Delta = \begin{bmatrix} \frac{\partial^2 \varphi_l^i}{\partial \left(w_1^i\right)^2} (P, w) & -\frac{\partial^2 \varphi_{l'}^i}{\partial \left(w_1^i\right)^2} (P, w) \\ \frac{\partial^3 \varphi_l^i}{\partial \left(w_1^i\right)^3} (P, w) & -\frac{\partial^3 \varphi_{l'}^i}{\partial \left(w_1^i\right)^3} (P, w) \end{bmatrix}
$$

and Γ is a 2×1 matrix with first component

$$
\frac{\partial^2 \varphi_{l'}^i}{\partial w_1^i \partial P_l} - \frac{\partial^2 \varphi_l^i}{\partial w_1^i \partial P_{l'}} + \left(\varphi_l^i - w_l^i\right) \frac{\partial^2 \varphi_{l'}^i}{\partial \left(w_1^i\right)^2} - \left(\varphi_{l'}^i - w_{l'}^i\right) \frac{\partial^2 \varphi_l^i}{\partial \left(w_1^i\right)^2}
$$

and second component

$$
\begin{aligned}
& \frac{\partial^3 \varphi_{l'}^i}{\partial \left(w_1^i\right)^2 \partial P_l} - \frac{\partial^3 \varphi_l^i}{\partial \left(w_1^i\right)^2 \partial P_{l'}} + \frac{\partial \varphi_l^i}{\partial w_1^i} \frac{\partial^2 \varphi_{l'}^i}{\partial \left(w_1^i\right)^2} \\
& - \frac{\partial \varphi_{l'}^i}{\partial w_1^i} \frac{\partial^2 \varphi_l^i}{\partial \left(w_1^i\right)^2} + \left(\varphi_l^i - w_l^i\right) \frac{\partial^3 \varphi_{l'}^i}{\partial \left(w_1^i\right)^3} - \left(\varphi_{l'}^i - w_{l'}^i\right) \frac{\partial^3 \varphi_l^i}{\partial \left(w_1^i\right)^3}
\end{aligned}
$$

Both Δ and Γ are identified, as they depend only on φ^i. Moreover, by Regularity, for some $w \in \mathbb{R}_+^L$ and $l, l' \in \{1, ..., L\}$, matrix Δ is invertible, which identifies ϕ_l^i and $\phi_{l'}^i$. For every other

$$
l'' \in \{1, ..., L\} \setminus \{1\}
$$

$\phi^i_{l''}$ is

$$\frac{\frac{\partial^2 \varphi^i_{l''}}{\partial w^i_1 \partial P_l} - \frac{\partial^2 \varphi^i_l}{\partial w^i_1 \partial P_{l''}} + \left(\varphi^i_l + \phi^i_l - w^i_l\right) \frac{\partial^2 \varphi^i_{l''}}{\partial \left(w^i_1\right)^2} - \left(\varphi^i_{l''} - w^i_{l''}\right) \frac{\partial^2 \varphi^i_l}{\partial \left(w^i_1\right)^2}}{\frac{\partial^2 \varphi^i_l}{\partial \left(w^i_1\right)^2}}.$$

Finally, ϕ^i_1 can be identified by Walras' law. ■

References

[1] Chiapori, P. Ekeland, I. Kubler, F and H. Polemarchakis. 2000. The Identification of Preferences from Equilibrium Prices. Core Discussion Paper 2000/24.

[2] Mas-Collel, A. Whinston M and J. Green. 1995. Microeconomic Theory. MIT Press.